Kinetic Analysis of the Thermal Isomerizations Linking 1-(*E*)-Propenyl-2-methylcyclobutanes with (*Z*)- and (*E*)-1,5-Octadiene and 3-Ethylhexa-1,5-diene

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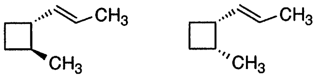
The gas-phase unimolecular thermal reactions of the cis and trans isomers of 1-(*E*)-propenyl-2-methylcyclobutane encompass a complex array of stereomutations, fragmentations, and structural isomerizations. The isomerizations linking 1-(*E*)-propenyl-2-methylcyclobutanes with (*Z*)- and (*E*)-1,5-octadiene and 3-ethylhexa-1,5-diene are governed by a nonhomogeneous system of differential equations which may be solved exactly and used to predict reaction rate profiles for the diene products. Comparisons between these predictions and kinetic data show that the retro-ene reaction of *cis*-1-(*E*)-propenyl-2-methylcyclobutane takes place with high stereoselectivity to yield (*Z*)-1,5-octadiene, which reacts further through Cope rearrangements to afford time-dependent mixtures of the three dienes.

Introduction

The thermal chemistry of vinylcyclobutane includes fragmentation to ethylene and 1,3-butadiene as well as structural isomerization to cyclohexene.[1−4] Vinylcyclobutanes substituted with alkyl groups such as the 1-(*E*)-propenyl-2-methylcyclobutanes **1** and **2** exhibit far more complicated reactivity.



1 **2**

Isomers **1** and **2** interconvert reversibly. They fragment to olefins and dienes, and alkyl-substituted conjugated diene fragmentation products may give secondary reactions through [1,5] hydrogen shifts. Retro-ene isomerizations may occur, and the products in turn suffer Cope rearrangements. Finally, additional products plausibly rationalized as arising from transient diradical intermediates may be formed.[5,6] Fortunately, most of these processes may be modeled kinetically through straightforward treatments, for they depend on easily integrated first-order differential equations or sets of homogeneous first-order differential equations which may be solved through common linear algebra techniques.[7,8]

During studies of the thermal chemistry of racemic and nonracemic versions of the cis and trans isomers of 1-(*E*)-propenyl-2-methylcyclobutane a less common kinetic situation was encountered, one which may be treated using an analysis based on a system of nonhomogeneous first-order differential equations.[6] Solutions of such sets of equations are well-known mathematically, but not generally covered in standard texts on reaction kinetics. The present work details how they may be solved in the specific case under investigation, and, by extension, to other similar situations posed by complex sets of multichannel unimolecular isomerizations. Predictions based on the kinetic treatment and comparisons with composition versus reaction time data reveal the stereochemical preference for the retro-ene isomerization of this substituted vinylcyclobutane.

Experimental Section

All kinetic data for gas phase thermal chemistry shown by 1-(*E*)-propenyl-2-methylcyclobutanes and by isomeric dienes were secured using a static reactor at 275 °C. Reaction mixtures were analyzed by capillary GC; all integrated peak intensities were referenced to an internal standard, cyclooctane.[6]

Results and Discussion

Kinetic and Stereochemical Issues. Kinetic studies of the thermal reactions shown by both racemic and nonracemic versions of *trans*-1-(*E*)-propenyl-2-methylcyclobutane (**1**) and *cis*-1-(*E*)-propenyl-2-methylcyclobutane (**2**) led to rate constants for all eight possible [1,3] carbon shift reactions giving rise to the four isomers of 3,4-dimethylcyclohexene.[5] The thermal stereomutations between **1** and **2** were followed, and concentration versus time functions for **1** and **2** were obtained.[5,6] For each of the four starting materials, these functions of the required form

$$\mathbf{1}(t) \text{ or } \mathbf{2}(t) = B_1\exp(-\lambda_1 t) + B_2\exp(-\lambda_2 t) \qquad (1)$$

were obtained through least-squares fits of experimental data; λ_1 and λ_2 were found to be 1.84×10^{-5} and 9.02×10^{-5} s^{-1}, whereas values of the constants B_1 and B_2 depended on the concentrations of **1**(0) and **2**(0).[6]

Retro-ene Reactions. The retro-ene reaction of 1-pentene gives ethylene and 1-propene; when there is a bridge between C3 and C4 of the 1-pentene moiety, the retro-ene process leads to an isomer, a nonconjugated diene. *cis*-1-Vinyl-2-methylcyclopropane gives (*Z*)-1,4-hexadiene;[9−12] 1-propenyl-2-methylcyclobutanes isomerize to 1,5-octadienes;[6,13] substituted 1-vinyl-2-alkylcyclopentanes afford 1,6-dienes.[14]

The stereochemical aspects of the isomerization of *cis*-1-vinyl-2-methylcyclopropane to 1,4-hexadienes are readily defined by

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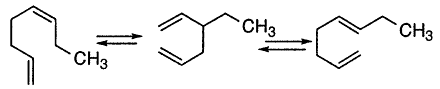
measuring the ratio of (*Z*) to (*E*) isomers in the product mixture.[9−11] Under normal experimental conditions, these isomeric dienes do not interconvert, and the product ratio is a constant at various reaction times.

Kinetically controlled product ratios from retro-ene isomerizations of 1-(*E*)-propenyl-2-methylcyclobutanes are not so easily determined because (*Z*)-1,5-octadiene (**3**) and (*E*)-1,5-octadiene (**5**) interconvert rapidly through Cope rearrangements and an intermediate structure, 3-ethylhexa-1,5-diene (**4**).



3 **4** **5**

This complexity was side-stepped in earlier work dedicated to finding the stereochemistry of [1,3] carbon shifts by treating [**3** + **4** + **5**] as a single entity.[6] One could then look for least-squares best-fit values for rate constants leading from **1** and from **2** to [**3** + **4** + **5**], a temporizing tactic valid whatever the rate constants connecting the three dienes. The equation

$$\mathrm{d}[\mathbf{3}+\mathbf{4}+\mathbf{5}]/\mathrm{d}t = (k_{13}+k_{15})\mathbf{1}(t) + (k_{23}+k_{25})\mathbf{2}(t) \quad (2)$$

was integrated and then used to estimate the two accessible rate constants. The values found for ($k_{23} + k_{25}$) were 1.97×10^{-5} and 1.88×10^{-5} s^{-1} for the two kinetic runs starting from racemic and nonracemic samples of **2.** Kinetic runs starting with racemic and nonracemic samples of **1** demonstrated that ($k_{13} + k_{15}$) was negligibly small.[6] Values of k_{23} and k_{25} individually were not secured.

Cope Rearrangements of 3-Ethylhexa-1,5-diene. The differential equations governing the reactions interconverting dienes **3**, **4**, and **5** are readily defined

$$\mathrm{d}\mathbf{3}/\mathrm{d}t = -k_{34}\mathbf{3}(t) + k_{43}\mathbf{4}(t) \quad (3)$$

$$\mathrm{d}\mathbf{4}/\mathrm{d}t = k_{34}\mathbf{3}(t) - (k_{43}+k_{45})\mathbf{4}(t) + k_{54}\mathbf{5}(t) \quad (4)$$

$$\mathrm{d}\mathbf{5}/\mathrm{d}t = k_{45}\mathbf{4}(t) - k_{54}\mathbf{5}(t) \quad (5)$$

This system of homogeneous first-order differential equations may be solved through well-known methods provided by matrix algebra.[7,8] They depend on the rate constant matrix **K**

$$\mathbf{K} = \begin{bmatrix} -k_{34} & k_{43} & 0 \\ k_{34} & -(k_{43}+k_{45}) & k_{54} \\ 0 & k_{45} & -k_{54} \end{bmatrix} \quad (6)$$

The eigenvalues η_i and eigenvectors $\mathbf{A}_i$ of matrix **K** afford solutions to eq 3−5, which may be expressed more economically as

$$\mathbf{x}' = \mathbf{K}\mathbf{x} \quad (7)$$

The solutions are given by

$$\mathbf{x}(t) = \Psi(t)\mathbf{x}(t=0) \quad (8)$$

where the solution matrix $\Psi(t)$ may be calculated from

$$\Psi(t) = \mathbf{A}\,\mathrm{diag}[\exp(-\eta_1 t), \exp(-\eta_2 t), \exp(-\eta_3 t)]\mathbf{A}^{-1} \quad (9)$$

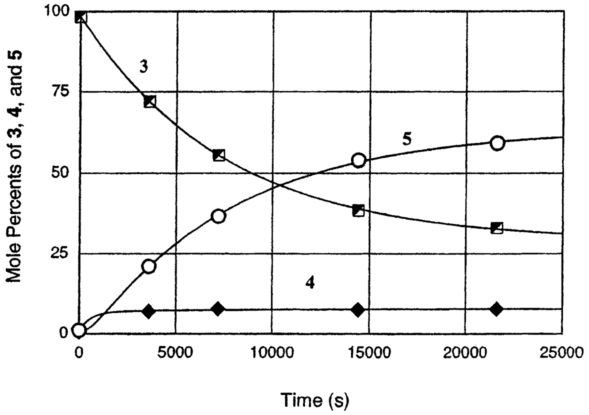


Figure 1. Time evolution of mole percent concentrations of dienes **3**−**5** at 275 °C starting from **3**: data points (Table 1) versus theory based on rate constants $k_{34} = 13$, $k_{43} = 50$, $k_{45} = 112$, and $k_{54} = 14$, all $\times\ 10^{-5}$ s^{-1}.

TABLE 1: Mole Percent Data for C_8H_{14} Cope Rearrangement Isomers Derived from (*Z*)-1,5-Octadiene (3) at 275 °C

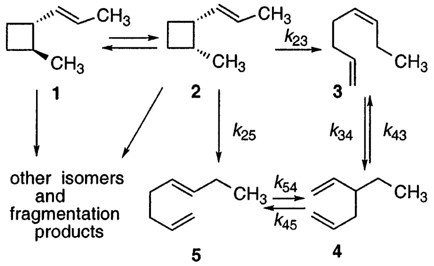
time (s)	3	4	5
0	98.3	0.6	1.1
3600	72.0	7.0	21.0
7200	55.5	7.8	36.7
14400	38.5	7.5	54.0
21600	33.1	7.7	59.2

The gas-phase thermal unimolecular equilibrations at 275 °C of **3**−**5** through Cope rearrangements were followed starting with the (*Z*) isomer **3**. The data obtained are summarized in Table 1.

The isomerization to **4** quickly proceeds toward an equilibrium mole percent value, whereas the equilibration between **3** and **5** by way of **4** requires longer reaction times. The same pattern is seen in similar reactions starting from (*Z*)-1,5-heptadiene.[15]

Initial estimates of concentrations at equilibrium based on the data set in Table 1 (30% **3**, 8% **4**, 62% **5**) gave $k_{43}/k_{34} \approx 3.8$ and $k_{45}/k_{54} \approx 7.8$. The data of Table 1 and least-squares best-fit calculations then led to $\eta_1 = 0$, $\eta_2 = 1.33 \times 10^{-4}$ s^{-1} and $\eta_3 = 1.76 \times 10^{-3}$ s^{-1} and then to all four rate constants ($k_{34} = 13$, $k_{43} = 50$, $k_{45} = 112$, and $k_{54} = 14$, all $\times\ 10^{-5}$ s^{-1}). The theory-based functions and estimated rate constants match the data reasonably well (Figure 1).

Kinetic Solutions for Nonhomogeneous Differential Equations. The kinetic situation posed by the retro-ene reaction of **2**(t) and the Cope rearrangements interconverting **3**−**5** is portrayed in the reaction scheme shown:



The corresponding differential equations which when solved provide theoretical expressions for the time evolution of dienes

3−**5** are

$$\mathrm{d}\mathbf{3}/\mathrm{d}t = -k_{34}\mathbf{3}(t) + k_{43}\mathbf{4}(t) + k_{23}\mathbf{2}(t) \qquad (10)$$

$$\mathrm{d}\mathbf{4}/\mathrm{d}t = k_{34}\mathbf{3}(\mathrm{t}) - (k_{43} + k_{45})\mathbf{4}(t) + k_{54}\mathbf{5}(t) \qquad (11)$$

$$\mathrm{d}\mathbf{5}/\mathrm{d}t = k_{45}\mathbf{4}(t) - k_{54}\mathbf{5}(t) + k_{25}\mathbf{2}(t) \qquad (12)$$

Now **2**(t) is a known function (eq 1) and all of the rate constants needed to solve this system of nonhomogeneous first-order differential equations are known, save for the balance between k_{23} and k_{25}. The sum ($k_{23} + k_{25}$) is known, so k_{23} may be expressed as $\alpha(k_{23} + k_{25})$ and k_{25} as $(1 - \alpha)(k_{23} + k_{25})$. The parameter α reflects the stereoselectivity of the retro-ene reaction of vinylcyclobutane **2**.

The nonhomogeneous equations may be expressed in matrix form

$$\begin{bmatrix}\mathbf{3}'\\ \mathbf{4}'\\ \mathbf{5}'\end{bmatrix} = \begin{bmatrix}-k_1 & k_{43} & 0\\ k_{34} & -(k_{43}+k_{45}) & k_{54}\\ 0 & k_{45} & -k_{54}\end{bmatrix}\begin{bmatrix}\mathbf{3}\\ \mathbf{4}\\ \mathbf{5}\end{bmatrix} + \begin{bmatrix}\alpha k\mathbf{2}(t)\\ 0\\ (1-\alpha)k\mathbf{2}(t)\end{bmatrix} \qquad (13)$$

Because **2**(t) is given by eq 1 and all parameters but α are known, the matrix equation could just as well be expressed as in

$$\begin{bmatrix}\mathbf{3}'\\ \mathbf{4}'\\ \mathbf{5}'\end{bmatrix} = \begin{bmatrix}-k_{34} & k_{43} & 0\\ k_{34} & -(k_{43}+k_{45}) & k_{54}\\ 0 & k_{45} & -k_{54}\end{bmatrix}\begin{bmatrix}\mathbf{3}\\ \mathbf{4}\\ \mathbf{5}\end{bmatrix} + \begin{bmatrix}\alpha kB_1 \exp(-\lambda_1 t)\\ 0\\ (1-\alpha)kB_1 \exp(-\lambda_1 t)\end{bmatrix} + \begin{bmatrix}\alpha kB_2 \exp(-\lambda_2 t)\\ 0\\ (1-\alpha)kB_2 \exp(-\lambda_2 t)\end{bmatrix} \qquad (14)$$

This formulation corresponds to the more abstract matrix equation

$$\mathbf{x}' = \mathbf{Kx} + \mathbf{f}_1(t) + \mathbf{f}_2(t) = \mathbf{Kx} + \mathbf{k}_1 \exp(-\lambda_1 t) + \mathbf{k}_2 \exp(-\lambda_2 t) \qquad (15)$$

where both $\mathbf{f}_1(t)$ and $\mathbf{f}_2(t)$ are just column vectors of constants times exponential functions. Such nonhomogeneous systems of equations may be solved using the method of undetermined coefficients, with $\mathbf{f}_1(t)$ and $\mathbf{f}_2(t)$ as forcing functions.[16] Matrix **A** was obtained while solving the complementary system of homogeneous equations, eq 7. For eq 14, particular solutions may be found by applying the relationships

$$\mathbf{c}_1 = -(\mathbf{A} + \lambda_1\mathbf{I})^{-1}\mathbf{k}_1 \qquad (16)$$

$$\mathbf{c}_2 = -(\mathbf{A} + \lambda_2\mathbf{I})^{-1}\mathbf{k}_2 \qquad (17)$$

$$\mathbf{x}\mathrm{p}_1(t) = \mathbf{c}_1 \exp(-\lambda_1 t) \qquad (18)$$

$$\mathbf{x}\mathrm{p}_2(t) = \mathbf{c}_2 \exp(-\lambda_2 t) \qquad (19)$$

The constant terms in the column matrixes $\mathbf{c}_1$ and $\mathbf{c}_2$ and the particular solutions $\mathbf{x}\mathrm{p}_1(t)$ and $\mathbf{x}\mathrm{p}_2(t)$ may be calculated directly from $\mathbf{f}_1(t)$ and $\mathbf{f}_2(t)$. In the present case, where **3**(0), **4**(0), and **5**(0) are all zero, the solution of eq 14 is

$$\mathbf{x}(t) = \mathbf{c}_1 \exp(-\lambda_1 t) + \mathbf{c}_2 \exp(-\lambda_2 t) - \Psi(t)(\mathbf{c}_1 + \mathbf{c}_2) \qquad (20)$$

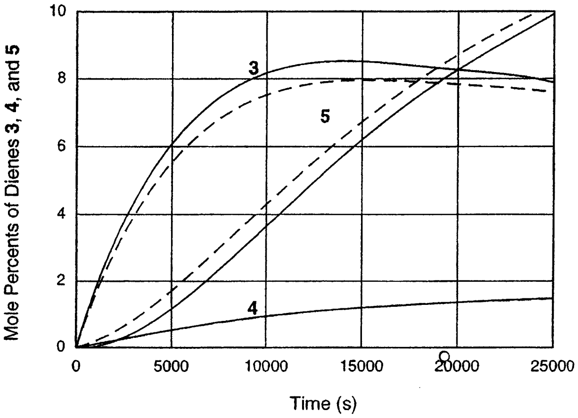


Figure 2. Theoretical mole percent concentration versus reaction time plots for dienes **3**−**5** at 275 °C, starting with racemic or nonracemic **2** (when α = 1.0, solid lines; for α = 0.9, dashed lines). The two calculated functions for **4** are essentially identical and the lines overlap.

Numerical solutions to this equation for any choice of α may be found easily with the aid of suitable software, such as the Linear Algebra routines provided in Maple 6.[17]

Predicted Concentration versus Time Kinetic Profiles. Figure 2 shows calculated mole percent concentration versus time plots for reactions starting from the cis isomer **2**. The theory-based predictions depend on parameters secured for **2**(t) starting from racemic and nonracemic samples of **2**.[6] When α was set at 1.0, the B_1, B_2, and k parameters are (0.48, 99.14, 1.97×10^{-5} s^{-1}) and (0.37, 99.63, 1.88×10^{-5} s^{-1}), respectively, and the calculated kinetic points are essentially identical, for the slight variations in input parameters are of scant consequence for the predictions. The calculated kinetic points were fit with polynominal functions to provide the theory-based predictions shown in Figure 2 (solid lines). When α was taken to be 0.9, the predicted concentrations were as shown by the dashed lines.

The function for isomer **4** shows no significant dependence on whether α is 1.0 or 0.9, whereas the plots for **3** and **5** differ somewhat, but not in any striking way. With k_{23} favored over k_{25} by a factor of 9, or 90, the mole percent function for **3** rises, reaches a maximum, and diminishes as the equilibration among **3**, **4**, and **5** proceeds. Experimental data roughly following the shapes of the functions shown in Figures 2 can be recognized qualitatively as indicating $k_{23} > k_{25}$, but in the absence of theory, no clear quantitative assessment may be made. The shapes depend on k_{23} and k_{25} and, importantly, on the Cope rearrangement rate constants.

Comparisons with Experimental Data. Figure 3 gives experimental data points[6] for kinetic runs starting with racemic and nonracemic samples **2** together with the theoretical predictions of Figure 2, for the case where α = 1.0. The fits are fairly good, though the scatter, about what one might expect for the analytical methods employed and the relatively small mole percent concentrations seen, is a distraction. Figure 4 presents the same experimental data, over the theoretical prediction of Figure 3, for the case where α = 0.9.

When the goodness of fit is gauged by calculating root-mean-square (rms) deviations, the α = 1 model is better. For the nonracemic sample of **2**, the rms values are 0.29% when α = 1.0 and 0.38% when α = 0.9. A very shallow minimum value for rms deviation was found for k_{23}/k_{25} = 20/1 (α = 0.95), 0.27%. For the data set starting from racemic **2**, the rms values were slightly higher, a reflection perhaps that more points at longer times were included.

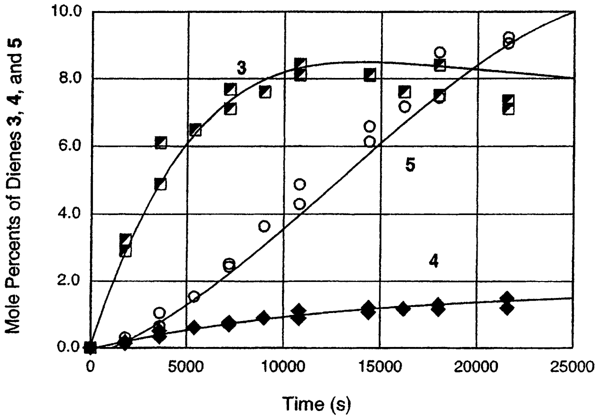


Figure 3. Theoretical concentration versus reaction time plots and experimental data for dienes **3**−**5** at 275 °C, starting with racemic or nonracemic **2**; α = 1.0.

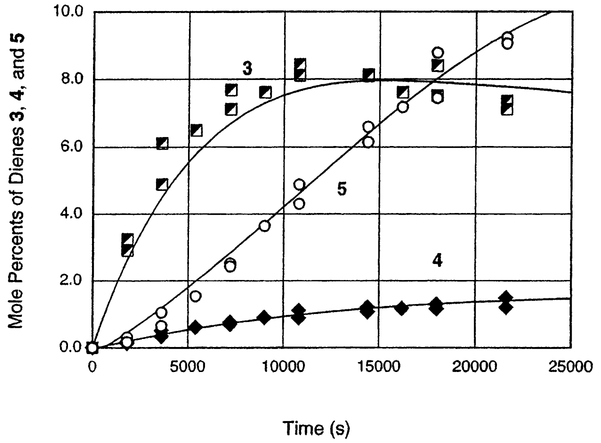


Figure 4. Theoretical concentration versus reaction time plots and experimental data for dienes **3**−**5** at 275 °C, starting with racemic or nonracemic **2**; α = 0.9.

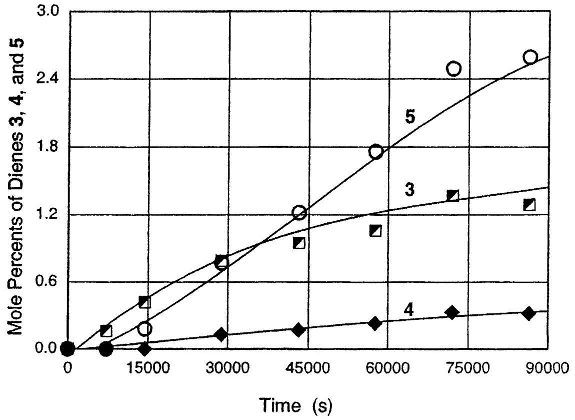


Figure 5. Theoretical concentration versus reaction time plots and experimental data for dienes **3**−**5** at 275 °C, starting with nonracemic **1**; α = 1.0.

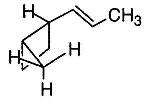
For reactions starting with nonracemic **1**, the formation and equilibration of isomers **3**−**5** take place over much longer reaction times, for the isomerization of **1** to **2** is substantially rate-limiting. Nevertheless, the theory-based predictions with B_1, B_2, and k parameters 89.39, 10.40, $1.88 \times 10^{-5}\ \mathrm{s}^{-1}$,[6] and α = 1.0, together with experimental data points, demonstrate a fine agreement (Figure 5).

Summary and Conclusions

The kinetic situation presented by the retro-ene reaction of **2** first loomed as an impasse,[6] for the nonhomogeneous system of first-order differential equations to be solved was of a sort not ordinarily encountered in standard chemical kinetics texts. Even books dedicated to linear algebra and interactive learning using powerful software for handling numerical details may neglect nonhomogeneous systems.[18] Yet the chemical situation is not particularly exceptional, and the mathematically correct treatment of the system of equations is known and easy to implement. The treatment outlined here may be readily adjusted to deal with similar sets of nonhomogeneous first-order differential equations, whenever the forcing functions are of the form $\mathbf{k}\exp(-\lambda t)$.

With the theory-based values for mole percent concentration versus time profiles, one can contrast experimental data versus theory-based predictions and judge the goodness of agreement, using first visual inspection and then, more tellingly, some statistical measure, such as root-mean-square deviation.

These comparisons for the set of reactions under consideration reveal that k_{23} is much greater than k_{25}; the ratio k_{23}/k_{25} is about 20. The combined uncertainties in the predictions, based on analytically precise mathematics and experimental data secured through other kinetic work, as well as on the inevitable scatter of experimental points recorded for the time evolution of **3**−**5** starting from **1** or **2**, precludes a sharper estimate. Yet one may conclude with confidence that the stereoselectivity of the retro-ene reaction of **2** strongly favors formation of (*Z*) isomer **3** rather than (*E*) isomer **5**. This sense of stereoselectivity has been demonstrated for several *cis*-1-vinyl-2-methylcyclopropanes.[9−12] The current work, the first report on the stereoselectivity of the retro-ene reaction of a vinylcyclobutane, suggests that this stereochemical preference is not dependent on the relatively rigid structural limitations of the cyclopropyl ring system. The far more conformationally supple cyclobutane-based homologous system exhibits the same stereochemical predilection. An "endo" transition structure (**6**) is again decisively preferred over the "exo" alternative (**7**).



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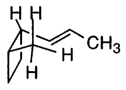

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